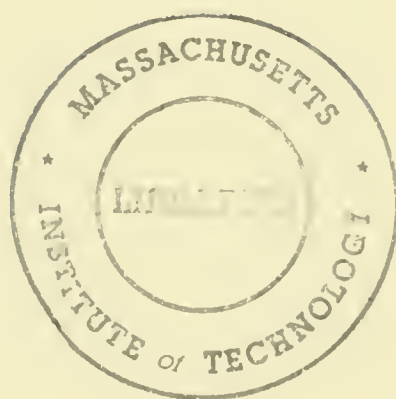


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MONOPOLISTIC PRICE ADJUSTMENT AND  
AGGREGATE OUTPUT

by

Julio J. Rotemberg

Massachusetts Institute of Technology

January 1981

WP 1186-81

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Julio J. Rotemberg<sup>\*</sup>

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\* Sloan School of Management, MIT. This paper is a revised version of Chapter II of my Ph.D. Dissertation which carries the same title. I am indebted to Alan Blinder, William Branson, Grégory Chow, Ed Green, Stephen Kealhofer and Robert Porter for helpful comments and suggestions.

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## I. Introduction.

There have been two major attempts to build a microeconomic foundation for the existence of fluctuations in aggregate output. The first one is associated with the work of Barro and Grossman [1976] in which prices are assumed to be fixed or else to follow some slow path towards their equilibrium values. The economic agents then proceed to maximize their objective functions subject to the fixed prices and to the rationing that naturally emerges in those markets in which supply is not equal to demand at the going prices. The second attempt is associated with the seminal papers by Lucas [1972, 1975]. He built an equilibrium model of the business cycle. In his model the prices are such that people succeed in carrying out the transactions they desire to carry out at these prices. However, agents are assumed to misperceive profitable opportunities. This is due to their inability to observe the value of aggregate statistics like the current price level and the level of the money supply. Instead monetary injections are momentarily perceived as good opportunities by everybody. Therefore they are followed by increases in aggregate output which dissipate as people learn about the old monetary injections.

This paper presents a new attempt at building a model which accounts for the existence of fluctuations in aggregate output in response to nominal disturbances, like an unpredicted injection of money into the economy. Like Lucas' model it is an equilibrium model. Economic agents maximize their objective functions taking the prices set by the other agents as given. They make the best use of current information in the computation of facts about their current and future economic environment. In fact, the producers, who in this model have market power, have full

information about the present. Namely, they know the prices charged by their suppliers, the price level, and the economy-wide level of nominal money balances. Furthermore they observe their demand and cost functions before they set their prices. These assumptions about the information available to producers sets this model apart from Lucas' and in my view constitute a theoretical advantage.

In this model it is the assumption that it costs resources to change prices, possibly due to the difficulties changing prices impose on consumers, that introduced the "rigidity" necessary for the existence of correlated responses in output to uncorrelated nominal shocks. The model is therefore a relative of the Barro-Grossman model in that it is the slow response of prices which is placed at the center of the explanation of business cycles. However, it parts company with the latter model in that the monopolies set their prices optimally given that it is costly to change them. This fact changes many of the qualitative features of the Barro-Grossman model.

The model of this paper has the added advantage of implying a positive correlation between the real wage and GNP. Instead, both the standard Keynesian and most variants of the Lucas model require the real wage to be relatively low when the output is relatively high. In the U.S. the real wage seems to have moved procyclically (see for instance Dunlop [1938], Tarshis [1939] and Blinder [1980]).

In the next section I discuss a monopolist's optimization problem when he is faced by specific demand and cost functions as well as an explicit stochastic environment. It will be costly for him to change prices, for reasons to be discussed, and therefore his optimization prob-

lem will be dynamic in nature. This is so because the price he sets today will affect the costs to changing prices tomorrow. In Section III, I construct a rational expectations equilibrium for an economy consisting solely of the monopolists discussed in Section II and of households. This rational expectations equilibrium is a stochastic process for the price level and for output that is driven by the stochastic shocks to the level of money balances and to the taste for real money balances. In section IV, I study some of the comparative dynamics of this rational expectations equilibrium. I show that when the money supply follows a random walk, output will be serially correlated. Furthermore I study the behavior of the equilibrium when there is a constant expected rate of monetary expansion and, alternatively, when the money stock is expected to change drastically in the future. In section V, I extend the model to include a labor market and show that real wages move procyclically. The last section contains some conclusions and a research agenda.

## II. Monopolies

The model consists of  $n$  monopolies indexed by  $i$ . Each one produces a distinct nonstorable good. In this section I derive the pricing rule that the monopolists will follow in the presence of a cost to changing prices. The analysis proceeds in two parts. First I study the maximization of the profits that result from sales, given a specific demand and cost functions, when changing prices is costless. Then the profits to be maximized become the profits from sales minus the costs of changing prices. The pricing rule is then derived from an optimal control problem.

The demand functions for the goods have the form:

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$$Q_{it}^d = A_i \left( \frac{P_{it}}{P_t} \right)^{b_i} \left( \frac{M_t}{P_t V_t} \right)^{d_i} \quad i = 1, 2, \dots, n \quad (1)$$

where  $Q_{it}^d$  is the quantity of good  $i$  demanded at time  $t$ ;  $A$ ,  $d$  and  $b$  are firm specific constants;  $P_{it}$  is the price of good  $i$  at time  $t$  which is set by the monopolist;  $P_t$  is the price level which is defined below;  $M_t$  is the economy-wide level of nominal money balances and  $V_t$  is a time varying taste parameter. Prices will change because  $V_t$  and  $M_t$  follow stochastic processes with normal disturbances. The monopolists are assumed to observe their demand function at time  $t$ ; therefore they observe the ratio of  $M$  to  $V$ . This is so because there is no firm specific uncertainty in this model. Furthermore the money supply is assumed to be a published statistic at time  $t$ . Therefore the monopolists observe both  $M$  and  $V$  in the current period.

The quantity demanded of any particular good depends not only on the relative price of the good but also on real money balances. This can be justified by introducing real money balances into the demanders utility functions. Note that as  $V$  changes a different level of real money balances leads to the same quantity of each good being demanded at constant relative prices. The second term of these demand functions can be thought of as a wealth effect.

The price level is a weighted geometric average of the  $n$  prices charged by the monopolists:

$$P_t = \left[ \prod_{i=1}^n (P_{it})^{h_i} \right]^{1/\sum h_i} \quad , \text{ where the } h_i \text{ are constants.} \quad (2)$$



This definition is convenient since it leads to a log-linear specification of the demand functions (1).

The cost functions of the monopolists are:

$$C_i(Q_{it}) = U_i P_t Q_{it}^2 / 2. \quad i = 1, 2, \dots, n. \quad (3)$$

Here  $C_i(Q_{it})$  is the cost to firm  $i$  of producing the quantity  $Q$  at time  $t$ ;  $U_i$  is a firm specific parameter which is assumed to be small and positive.

Marginal costs therefore increase with output. Since the price level includes the price of the own good, goods can be used in the production of themselves. Here I assume that, when purchasing its own output for use as an input, the productive side of the firm is charged a price which is equal to the price the firm charges its other customer times a fixed discount factor. Then the productive side of the firm proceeds to minimize costs. This assumption would not be necessary if good  $i$  were not productive in the production of good  $i$ . As long as the weight of any given good is small in the price level this assumption will not have any important consequences. It also appears to be of descriptive value. Two other points concerning this production function deserve to be mentioned. In this economy there are no nonproduced factors (like labor, for instance). However, the quantity of output that can be produced is bounded. There is a production possibility frontier. Once the net production of  $(n-1)$  goods is given there is a maximum of the  $n$ 'th good that can be produced. Furthermore, this cost function leads any cost minimizing firm to demand function for factors such that the demand for factor  $j$  depends negatively on the ratio of the price of factor  $j$  to the price level.

The monopolists take every other firm's prices as given. Therefore, prices will be the strategic variable in the Nash equilibrium of section

III. This equilibrium concept does not allow the monopolists to be fully rational. When a monopolist cuts its price the cost functions of all other monopolists are affected. This leads all the other monopolists to cut their prices which, in turn, leads to lower costs to the monopolist that originally considered a price cut. This secondary effect of a price change is not taken into account by these monopolists when they are deciding on a price for their product.<sup>1/</sup>

The first order conditions for the maximization of nominal profits require that  $P_{it}$  be equal to  $P_{it}^*$  where:

$$P_{it}^* = P_t \theta_i U_i Q_{it} \quad i = 1, 2, \dots, n \quad (4)$$

with

$$\theta_i = \frac{b_i - (1/2 + b_i - d_i)h_i / \sum h_i}{b_i - 1 - (b_i - d_i)h_i / \sum h_i} \quad (5)$$

If instead, as one would expect, the monopolist sought to maximize "real" profits, i.e. profits deflated by the price level, the first order conditions would be of the form of (4) with  $\theta_i$  now equal to:

$$\theta_i = \frac{b_i - (b_i - d_i)h_i / \sum h_i}{b_i - 1 - (b_i - 1 - d_i)h_i / \sum h_i} \quad (5')$$

In either case<sup>2/</sup>  $\theta_i$  has to be larger than one for the monopolist to be at a profit maximum; this condition reduces to  $b_i$  being larger than one if the effect of the own price on the price level is negligible, if  $h_i$  is very small compared with  $\sum h_i$ .

In the remainder of the paper I assume that the monopolist is concerned with the maximization of "real" profits. This is the more

reasonable assumption when the monopolist faces a tradeoff between profits in periods in which the price level is low versus profits in periods in which the price level is high.

I now take a second order approximation of profits around  $P_{it}^*$ .

$$\pi(P_{it}) = \pi(P_{it}^*) + \frac{d\pi}{dP_{it}} \cdot (P_{it} - P_{it}^*) + \frac{1}{2} \frac{d^2\pi}{dP_{it}^2} \cdot (P_{it} - P_{it}^*)^2 \quad (6)$$

where  $\pi$  are real profits from sales.

However,  $d\pi/dP_{it}$  is equal to zero at  $P_{it}^*$  and:

$$\frac{d^2\pi}{dP_{it}^2} = - \frac{2U_1 Q_{it}^2}{P_{it}^2} [(b_i - (b_i - d_i)/\Sigma h_i)^2 + (b_i - (b_i - d_i)/\Sigma h_i)] \quad (7)$$

which is negative.

Letting lower case letters denote the natural logarithm of their respective upper case letters one can further approximate:

$$(P_{it} - P_{it}^*)^2 \cong P_{it}^2 (p_{it} - p_{it}^*)^2 \quad (8)$$

Therefore (6) can be rewritten as

$$\pi(P_{it}) = \pi(P_{it}^*) - k_{it} (p_{it} - p_{it}^*)^2 \quad (6')$$

$$\text{with } k_{it} = P_{it}^2 \frac{d^2\pi}{dP_{it}^2}.$$

In principle  $k_i$  is not constant over time since it depends on the level of output that would be supplied if the price were set equal to  $P_{it}^*$ . This level of output is proportional to  $(M_t/PV_t)^{d_i/(1+b_i)}$ . Since real money

balances will move procyclically in Section III the parameter  $k_i$  ought to move procyclically. However, the variation of  $k_i$  will be neglected. As shown in the Appendix the error in the evaluation of profits due to this approximation is of the same order of magnitude as the error due to the neglect of the influence on profits of the higher powers of  $(p_{it} - p_{it}^*)$ . It is therefore a reasonable approximation for small variations of  $m$  and  $v$ , the variables that drive the system.

Eq. (6') gives the profits that would be forthcoming if changing prices were costless. The monopolist is actually concerned with the maximization of the discounted value of the above profits minus the cost of price changes.—<sup>3/</sup>

As has often been pointed out (Barro (1972), Mussa (1976), Sheshinski and Weiss (1977)), changing prices is costly for two reasons: First there is the administrative cost of changing the price lists, informing dealers, etc. Secondly, there is the implicit cost that results from the unfavorable reaction of the customers to large price changes.

While the administrative cost is a fixed cost per price change, the second cost can be a different function of the magnitude of the price change. In particular, customers may well prefer small and recurrent price changes to occasional large ones. This is what is implicitly assumed in this paper as I make the costs to changing prices a function of the square of the price change.—<sup>4/</sup>

The assumption that price changes are perceived to be costly is not only plausible on theoretical grounds but can also be justified by its power to explain the pricing behavior of manufacturing firms. Their prices change

seldom when compared with the prices for goods that are sold on competitive centralized markets like agricultural goods, stocks, etc.

When changing prices is costly, the monopolist might consider maintaining the old price while rationing some consumers. This is ruled out in this model by implicitly assuming that the costs to rationing consumers are enormous. These costs, too, are related to the reputation of the individual monopolists.

The existence of costs to changing prices alters the qualitative form of the monopolist's maximization problem. In the absence of these costs he chooses his prices in each period according to rule (4). In their presence today's decisions (prices) affect tomorrow's profits. This is so because it will be costly to charge tomorrow a price different from the one the firm decides to charge today. Therefore at time  $\tau$  the best price for  $\tau$  must be determined by maximizing the expected present discounted value of real profit given by:

$$E_{\tau} \left( \sum_{t=\tau}^{\infty} \rho^t [\pi(p_{it}^*) - k_i (p_{it} - p_{it}^*)^2 - c_i (p_{it} - p_{it-1})^2] \right) \quad (9)$$

where  $c_i$  is a firm specific constant. I now impose a relation between  $c_i$  and the other firm specific parameters ( $A_i$ ,  $b_i$  and  $V_i$ ) by assuming that:

$$c_i/k_i = c \quad i = 1 \dots, n$$

where  $c$  is a constant. This assumption is required to make the equilibrium of this economy tractable.

The monopolist's problem can now be rewritten as:



$$\text{Min } E_{\tau} \left\{ \sum_{t=\tau}^{\infty} \rho^t [(p_{it} - p_{it}^*)^2 + c (p_{it} - p_{it-1})^2] \right\} \quad (10)$$

(10) is a standard optimal control problem. The control variables are the price changes from  $\tau$  onwards. The state variables are the  $p_{it}$ 's and the  $p_{it}^*$ 's. Problems of the form of (10) have been solved by numerous authors. See Kennan (1979) for an extensive list of references from the investment literature. Sargent (1979) also solves problem of this type. His method of solution is employed here.

What is conceptually different about equation (10) is that here it is costly to adjust prices. Instead, the literature on costs of adjustment has concerned itself with competitive firms that take prices as given and have costs to changing quantities (employment, the capital stock, etc.).

The minimization of (10) leads to the following first order conditions:

$$p_{i\tau/t}^i - (1/\rho c + 1/\rho + 1)p_{i\tau/t-1}^i + (1/\rho)p_{i\tau/t-2}^i = -(1/\rho c)p_{i\tau/t-1}^{*i} \\ t = \tau + 1, \tau + 2, \tau + 3, \dots \quad (11)$$

where the superscript  $i$  indicates that these are expectations formed by firm  $i$ .  $p_{i\tau/t}^i$  is the expectation formed by firm  $i$  at  $\tau$  of the price  $p_{it}$  while  $p_{i\tau/t}^{*i}$  is the expectation formed by firm  $i$  at  $\tau$  of the price  $p_{it}^*$ .

The minimization of (12) also involves a transversality condition which takes the place of the end point condition in a finite horizon control problem:

$$\lim_{t \rightarrow \infty} (p_{i\tau/t}^i - p_{i\tau/t}^{*i}) + c(p_{i\tau/t}^i - p_{i\tau/t-1}^i) = 0 \quad (12)$$

Note that the model satisfies the conditions for first-period certainty equivalence. At time tau the monopolist holds an expectation for the future values of  $p_{it}^*$ . He decides on a path for his own prices given these expectations. These are the prices he expects to charge in the future. The solution of equation (11) is a path for the expected prices of the good  $i$  as a function of the path of the expected values of  $p_{it}^*$  and of  $p_{i\tau-1}$  which is given.

Using the lag operator  $L$  such that  $Lp_{i\tau/t}^i = p_{i\tau/t-1}^i$  (11) becomes:

$$(1 - (1/\rho c + 1/\rho + 1)L + 1/\rho)L^2)p_{i\tau/t}^i = -\frac{L}{\rho c} p_{i\tau/t}^* \quad (13)$$

And, factoring the expression on the LHS of (13)

$$(1 - \alpha L)(1 - \beta L)p_{i\tau/t}^i = -\frac{L}{\rho c} p_{i\tau/t}^* \quad (14)$$

$$\text{where } \alpha + \beta = 1/\rho c + 1/\rho + 1 \quad (15)$$

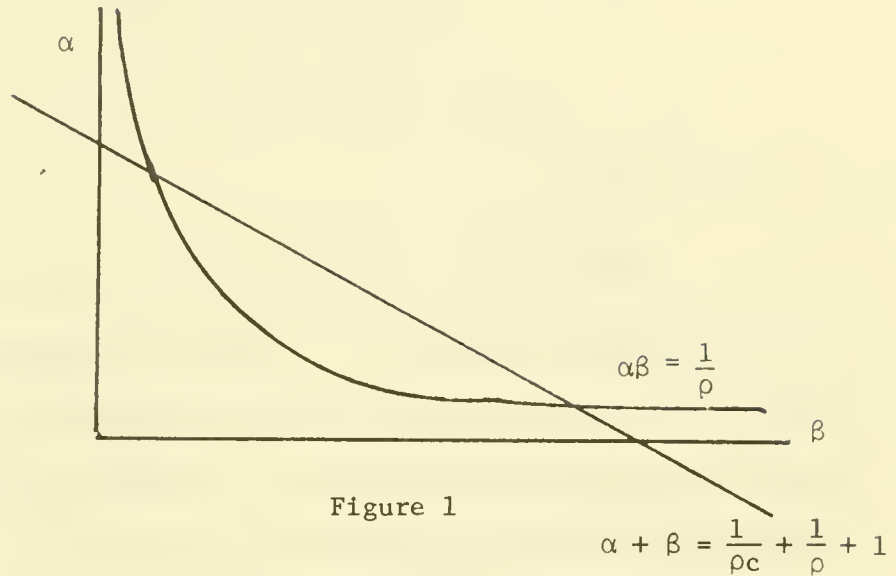
$$\alpha\beta = 1/\rho$$

and therefore

$$(1 - \alpha)(1 - \beta) = -\frac{1}{\rho c} \quad (16)$$

Since the sum and the product of the two roots are both positive, the roots are positive. Furthermore the fact that both  $\rho$  and  $c$  are positive implies that one of the roots is greater than one while the other is smaller than one.

This can be seen graphically:



The intersections of the two curves give the roots  $\alpha$  and  $\beta$ .

The equations for the sum and the product of the two roots make it clear that they are symmetric about one. Let  $\alpha$  be the smallest root. In Figure 1 one can see that a decrease in  $c$  moves the straight line to the right thereby increasing  $\beta$  and reducing  $\alpha$ . Instead an increase in  $\rho$  has ambiguous effects since it shifts both curves to the right.

To obtain the values of the sequence of  $p_{i\tau}/t$  for (14) two end-point conditions are needed. One initial condition is the value of  $P_{i\tau/\tau-1}$  which is given to the firm by its own history. The other end point condition is given by (12). This transversality condition says that the firm expects to charge a price close to  $p_{i\tau/t}^*$  in the far future. It requires, as shown in Sargent (1979) that the "unstable" root  $\beta$  be used to solve for  $p_{i\tau/t}$  forwards in time. In other words, it will be

satisfied as long as both sides of (14) are divided by  $(1 - \beta L)$  yielding:

$$p_{i\tau/t}^i (1 - \alpha L) = \frac{1}{\beta \rho c (1 - 1/L\beta)} p_{i\tau/t}^{*i} \quad (17)$$

or equivalently:

$$p_{i\tau/t}^i = \alpha p_{i\tau/t-1}^i + \frac{1}{\beta \rho c} \sum_{j=0}^{\infty} \left(\frac{1}{\beta}\right)^j p_{i\tau/t+j}^{*i} \quad (18)$$

(18) determines the price at  $\tau$ . However, in general the expectation held at  $\tau + 1$  on the sequence  $p_{it}^*$  will differ from those held at  $\tau$ . Therefore, at  $\tau + 1$ , firms will set prices for  $\tau + 1$  that maximize expected profits conditional on information available to them at  $\tau + 1$ . The prices that will be chosen for  $(\tau + 1)$  can be computed from (18) by replacing  $\tau$  by  $(\tau + 1)$ . In general the equation that determines the price charged by firm  $i$  at  $t$  is:

$$p_{it}^i = \alpha p_{it-1}^i + \frac{1}{\beta \rho c} \sum_{j=0}^{\infty} \left(\frac{1}{\beta}\right)^j p_{it/t+j}^{*i} \quad (19)$$

As mentioned earlier the existence of a cost to changing prices makes the monopolist change his prices slowly (therefore prices at  $t$  depend on prices at  $t-1$ ). These costs also lead monopolists to take into account the expected future optimal price when deciding on the current price so as to avoid costs of changing prices in the future. The price that the  $i^{\text{th}}$  monopolist charges today is a weighted average of the price he charged yesterday and the prices he would like to charge in all future periods if there were no cost to changing prices. Notice that if

$p_{it}^*$  follows a random walk its expectations at  $t$  for the infinite future is just its  $p_{it}^*$  and equation (19) reduces to:

$$\begin{aligned} p_{it}^i &= p_{it-1}^i + \frac{1}{\beta \rho c} \sum_{j=0}^{\infty} \left(\frac{1}{\beta}\right)^j p_{it}^{i*} = \alpha p_{it-1}^i + \frac{1}{\beta \rho c} \frac{\beta}{\beta - 1} p_{it}^{i*} \\ &= \alpha p_{it-1}^i + (1 - \alpha) p_{it}^{i*} . \end{aligned} \quad (20)$$

The last equality is obtained by using (16). Equation (20) is just the partial adjustment equation often used in empirical research. An increase in  $c$ , the cost to changing prices, was shown to increase  $\alpha$ . Thus, such an increase slows the adjustment of prices towards their long-run value, as expected. On the other hand, a change in the discount rate has an ambiguous effect on this speed of adjustment. While a decrease in  $\rho$  makes it relatively cheaper to change prices in the future (thereby leading to slower adjustment) it also penalizes the monopolist relatively more for current deviations of  $p_i$  from  $p_i^*$ .

### III. Equilibrium

In this section I construct the equilibrium of an economy consisting solely of monopolists and of the households that own the monopolies and spend the profits on output. In this economy the monopolists perceive that changing prices is costly. This equilibrium is constructed in three subsections.

In the first I discuss the form of Walras Law in this economy. In the second subsection I construct the equilibrium that would prevail in the absence of costs to changing prices. This is done for two reasons: First,



this will be the appropriate "long-run" equilibrium concept when changing prices is costly. Second, I will show that this equilibrium is strictly inside the production possibility frontier and that allocations with more output (which will emerge when changing prices becomes costly) are feasible.

In the third subsection the equilibrium for an economy in which price changes are perceived to be costly is derived under rational expectation. It takes a form similar to (19). It is a difference equation for the price level which is driven by the level of nominal money balances and by the taste parameter  $v$ .

### III.A. Walras Law

The output of the monopolists goes to two sectors. First, some of the output is demanded by other firms as an input. Second, some of it is demanded by households. Households are given the profits of the monopolists and it is these that they use to purchase goods. Let  $D_{it}$  be the demand by households of good  $i$  at time  $t$ ,  $F_{ijt}$  be the quantity of good  $j$  demanded by firm  $i$  in period  $t$ . As assumed,  $D_{it}$  depends on real money balances at time  $t$ . The households also have a demand for the  $n+1^{st}$  good, money. This demand is written as a demand for flows of additional amounts of nominal money above the previous stock of money:

$$M_t^d - M_{t-1} = M_t^s - M_{t-1} + \sum_i \Pi_i - \sum_i P_{it} D_{it} = Y_t - C_t^d \quad (21)$$

Here  $Y_t$  is nominal income of households in period  $t$  and  $C_t^d$  is the amount of consumption demanded in period  $t$ . The increase in nominal money

supplied is part of the income of the households since here money is assumed to be distributed by a helicopter and held only by households from one period to the next. Note that, since the demand for goods depends positively on the stock of money deflated by the price level, the excess demand for money in flows depends negatively on the level of real money balances.

The more usual demand for money is specified as a stock demand and it is the excess of real money balances over the desired stock of real money balances that affects the amount of goods demanded. Conceptually there is no difference between the usual approach and my own in a model in which money is the only store of value and there is no reshuffling of the individuals' portfolios. It is only the functional form of the demand for goods that is affected.

Equation (21) when the desired quantities are replaced by the actual quantities transacted is also the households budget constraint. In this model there is no rationing so there is no difference between the ex ante equation (21) and the ex post budget identity.

The individual firm's budget constraint is:

$$\Pi_{it} = P_{it}Q_{it} - \sum_j P_{jt}F_{ijt} \quad i = 1, 2, \dots, n. \quad (22)$$

The equilibrium condition in the goods market is that:

$$Q_{it} = D_{it} + F_{it} = D_{it} + \sum_j F_{jit} \quad i = 1, 2, \dots, n. \quad (23)$$

Replacing (23) in (22) and adding over all firms:

$$\sum_i \Pi_{it} = \sum_i P_{it} Q_{it} - \sum_{ij} P_{jt} F_{ijt} = \sum_i P_{it} D_{it}. \quad (24)$$

Therefore equilibrium in the  $n$  goods markets implies equilibrium in the money market. This can be seen by replacing for total profits in equation (21). I will therefore consider only the goods markets when computing the equilibrium of this economy.

### III.B. Equilibrium without cost to changing prices

Taking logarithms on both sides of (4):

$$p_{it} = p_t + (\theta_i + u_i + a_i)/(1 + b_i) + d_i (m_t - p_t - v_t)/(1 + b_i) \quad (25)$$

where again logarithms are denoted by lower case letters.

The equilibrium price level can easily be computed by weighting the above equations by  $h_i$  and summing:

$$\sum_i h_i p_{it} = p_t \sum_i h_i + \sum_i \frac{(\theta_i + u_i + a_i) b_i}{1 + b_i} + \left[ \sum_i \frac{d_i h_i}{1 + b_i} \right] (m_t - p_t - v_t) \quad (26)$$

The first term on the RHS is equal to the LHS by the definition of the price level. Therefore:

$$m_t - p_t - v_t = - \left[ \sum_i \frac{h_i(\theta_i + u_i + a_i)}{1 + b_i} \right] / \left[ \sum_i \frac{d_i h_i}{1 + b_i} \right] \quad (27)$$

Equation (27) says that real money balances depend on all the parameters of the model. It also can be used to see whether an expansion in the quantity of money  $m_t$  or a change in the taste for real money balances  $v_t$  has any effect on real output.

I will use as an index of aggregate output the sum of the logarithms of the output of the  $n$  goods.

$$q_t = \sum_i q_{it} \quad (28)$$

And using (1) in the definition (28) output demanded is:

$$q_t = \sum_i a_i + (m_t - v_t - p_t) \sum_i d_i \quad (29)$$

The equilibrium condition (27) assures that  $m_t - p_t - v_t$  is unaffected by changes in either the level of money balances or the taste for real money balances. Therefore, given the aggregate demand equation (29), aggregate output will not respond to such changes.<sup>5/</sup> This economy is neutral. Increases in nominal money balances are matched by increases in the price level. The "real" side of the economy, namely outputs and relative prices, remains unchanged. The neutrality of this economy depends on both the monopolists' knowledge of their input prices, demand<sup>6/</sup> and cost functions, and on the absence of any costs to changing prices.

Another point deserves to be made about these static equilibria. The competitive equilibrium can be computed in exactly the same way except that

$\theta_i$  is in this case equal to one. That is competitors set price equal to marginal cost. Then  $\theta_i$  is zero and the weighted sum of the  $\theta_i$  that appears in (27) is zero. On the contrary the  $\theta_i$ 's must be positive when the economy consists of monopolists. Therefore real money balances are larger when the economy is competitive. Larger real money balances lead to a larger aggregate output by (29). Therefore the monopolists produce less than the competitors in equilibrium. Recall that the competitive equilibrium is on the production possibility frontier. Therefore the monopolistic equilibrium without costs to changing prices is strictly inside the production possibility frontier.<sup>7/</sup> This means that allocations with more output than the one corresponding to this static equilibrium are feasible. When analyzing the behavior of the economy with costly price adjustment it will be apparent that sometimes the price level will be below the price level that would prevail in the absence of such costs and at other times it will be above. As long as the price level is not too far below the price level the monopolists would desire the allocation will be technologically feasible.

### III.C Equilibrium with costly price adjustment

We saw in Section II that a monopolist who faces a cost to changing prices will choose a pricing rule like (19) in which he takes the future into account. In fact, at each point in time the monopolist knows the whole sequence of prices that he will charge from that point on as long as he doesn't have to revise his estimates for the price level, for the stock of money and for  $v_t$ . The path of prices the monopolist expects to charge is the path of prices he will actually charge only when there is



no uncertainty. Denote the expectation held at  $t$  by firm  $i$  of the values at  $t + j$  of the price level, the level of money balances and the taste for money balances  $v$  by  $p_{t/t+j}^i$ ,  $m_{t/t+j}^i$  and  $v_{t/t+j}^i$  respectively. Then at time  $t$  the equilibrium price level will satisfy:

$$p_t = \alpha p_{t-1} + \frac{1}{\beta \rho c} \sum_i \left( \frac{1}{\beta} \right)^j \left\{ \frac{\sum_i h_i p_{t/t+j}^i}{\sum_i h_i} + \frac{\sum_i h_i s_i}{\sum_i h_i} + \frac{1}{\sum_i h_i} \sum_i \left( \frac{d_i h_i}{1 + b_i} \right) (m_{t/t+j}^i - p_{t/t+j}^i - v_{t/t+j}^i) \right\} \quad (30)$$

where

$$s_i = (\theta_i + u_i + a_i)/(1 + b_i).$$

The minimal requirement for  $p_t$  to be an equilibrium is that it agrees with the price level monopolists perceive today and that  $p_{t/t} = p_t$ . I will also assume that all  $n$  monopolists have homogeneous expectations about all the random variables in the system  $(p_t, m_t, v_t)$ . Furthermore I will require these expectations to be "rational" or, in other words, to be consistent with the model of this paper. This requires that the monopolists know the parameters of the model. At time  $t$  the monopolists expect to set future prices according to a rule of the form of (18). Therefore, if one knew how these monopolists set their prices as well as the price level and level of money balances they expect to prevail, one could compute the expected future price levels (i.e. the price level that would be forthcoming if the monopolists were not surprised by the course of future events) as follows:

$$p_{t/t+k} = \alpha p_{t/t+k-1} + \frac{1}{\beta \rho c} \sum_j \left( \frac{1}{\beta} \right)^j \{ p_{t/t+k+j} + S + D(m_{t/t+k+j} - p_{t/t+k+j} - v_{t/t+k+j}) \} \quad (31)$$

where

$$S = \frac{\sum_i h_i s_i}{\sum_i h_i} \quad D = \left( \sum_i \frac{h_i d_i}{1 + b_i} \right) \sum_i h_i \quad (32)$$

and  $m$  denotes the expectation held in common by all firms at  $t$  of the value  $t/t+k$  of  $m$  at  $t+k$ .

A rational expectations equilibrium is, in this model, a sequence of expected price levels that enter the RHS of equations (31) and (30), and which are equal to the LHS of the corresponding equation (31).<sup>8/</sup> Notice that this sequence of expected price levels is conditional on the common beliefs about the expectation of  $m_t$  and  $v_t$ . It must also be noted that if the consistency of the expectations was not imposed, almost any price level could be the equilibrium price level at time  $t$ . That is, if one is allowed to pick at will the values of  $(m_{t/t+j}^i, p_{t/t+j}^i, v_{t/t+j}^i)$ , then one can also pick almost at will a  $p_t$  that satisfies equation (30).

The rational expectations equilibrium can now be computed as the solution to equation (31). It is a difference equation for the expected price level that is driven by the expected levels of money balances and of the desire to hold real money balances.

By using the lag operator,  $L$ , such that:

$$L p_{t/k} = p_{t/k-1}$$

one can rewrite equation (31) as:

$$\left[ \frac{1-D}{\beta \rho c} \frac{1}{1-1/\beta L} - (1-\alpha L) \right] p_{t/t+k} = \frac{D}{\beta \rho c} \frac{1}{1-1/\beta L} \cdot (m_{t/t+k} - v_{t/t+k} + S/D) \quad (33)$$

Multiplying both sides of (33) by  $(\beta L - 1)$ :

$$\left[ 1 - \left( \frac{D-1}{\rho c} + \alpha + \beta \right) L + \alpha \beta L^2 \right] p_{t/t+k} = - \frac{LD}{\rho c} [m_{t/t+k} - v_{t/t+k} + S/D] \quad (34)$$

Factoring the LHS:

$$(1 - \gamma L)(1 - \delta L) p_{t/t+k} = - \frac{LD}{\rho c} [m_{t/t+k} - v_{t/t+k} + S/D] \quad (35)$$

where

$$\delta + \gamma = \alpha + \beta + \frac{D-1}{\rho c} = 1/\rho + 1 + D/\rho c$$

and

$$\delta \gamma = \alpha \beta = 1/\rho$$

and therefore:

$$(1 - \delta)(1 - \gamma) = - D/\rho c \quad (36)$$

Once again the two roots are positive; one is larger and the other smaller than one. A similar diagram to Figure 1 can be used to study the change in the roots as the parameters  $D$ ,  $c$ , and  $\rho$  change. I will call the root that is smaller than one,  $\gamma$ . As either  $c$  decreases or  $D$  increases,  $\gamma$  goes down while  $\delta$  goes up. As before, to satisfy the transversality condition (12), I solve forward with the unstable root which is equivalent to dividing both sides of equation (36) by  $(1 - \delta L)$ . This yields:

$$p_{t/t+k} = \gamma p_{t/t+k-1} + \frac{D}{\delta \rho c} \sum_{j=0}^{\infty} \left( \frac{1}{\delta} \right)^j (m_{t/t+k+j} - v_{t/t+k+j} + S/D) \quad (37)$$

As can be seen from the definition of  $S$  and  $D$  together with (27),  $(m_{t/t+k} - v_{t/t+k} + S/D)$  is the price level that would be expected at time  $t$  to prevail at time  $t+k$  if there were no costs to changing prices. So, the price level at  $t+k$  is expected to be a function of the previous price level and of the ulterior "desired" price levels. In other words, the price level is expected to slowly adjust towards the price levels that would prevail in the absence of costs to changing prices.

Note that  $\gamma$  is not necessarily equal to  $\alpha$ . This is so because the price level at  $t$  appears both on the RHS of equation (31) and on the LHS as long as  $D$  is different from one. Therefore the amount of the price level of the previous period contained in today's price level is a function of how strongly "excess" real money balances affect the demand for goods. The higher  $D$ , the higher the effect of wealth on demand, the lower  $\gamma$  and therefore the faster the adjustment to the "desired" price level given the values of the discount rate and of the cost to changing prices that affect  $\alpha$ .

Remembering that the price expected at  $t$  for  $t$  is the actual price level at  $t$ , the path for  $p_t$  is:

$$p_t = \gamma p_{t-1} + \frac{D}{\delta \rho c} \sum_{j=0}^{\infty} \left( \frac{1}{\delta} \right)^j (m_{t/t+j} - v_{t/t+j} + S/D) \quad (38)$$

The rest of this chapter is devoted to the analysis of (38) along with the path for output that it implies.<sup>9/</sup>

#### IV. Comparative Dynamics

##### Case 1     $m$ and $v$ follow random walks

The first case to consider is the one in which  $m$  and  $v$  follow a random walk without drift. In this case, at the time the monopolist sets his price, he expects the whole future of  $(m-v)$  to remain at its current level. As before, he observes his demand function as well as the

price level which is part of his cost function. Since he also knows the values of the fixed parameters ( $u$ ,  $a$ ,  $\Theta$ ), he can infer the current value of  $(m-v)$ . Therefore:

$$p_t = \gamma p_{t-1} + \frac{D}{\delta \rho c (1 - 1/\delta)} (m_t - v_t + S/D) = \gamma p_{t-1} + (1 - \gamma) (m_t - v_t + S/D) \quad (39)$$

where the last equality is obtained by using (36).

At  $t$ ,  $(m_t - v_t + S/D)$  is the level that would prevail if there were no costs to changing prices. I denote this price level that fulfills the conditions of (27) by  $p_t^*$ . If  $m$  and  $v$  did indeed remain constant from  $t$  onwards, the price level would converge to  $p_t^*$ . Therefore, it is natural to call  $p_t^*$  the long-run equilibrium price level for the economy with a cost to changing prices.

Equation (39) can be rewritten as:

$$p_t = \gamma p_{t-1} + (1 - \gamma) p_t^* \quad (40)$$

This equation is of the partial adjustment type and it states that the price level is a weighted average of the previous price level and of the long-run equilibrium price level. What is interesting about this equation is that it is precisely the equation used in the MPS model as described by De Menil and Enzler [1972] to describe the dynamics of the price level. This equation has come under the attack of McCallum [1979] for being ad hoc.

While the derivation that leads to (40) makes this criticism less valid, it must be said that the dynamics of the U.S. money supply cannot be described by a random walk.

To analyze the behavior of output when  $m$  and  $v$  follow a random walk we need to use the definition of aggregate output, (29), and replace the equation describing the path of the equilibrium price level, (30), into it:

$$\begin{aligned} q_t &= a + \sum_i d_i [m_t - v_t - \gamma p_{t-1} - (1 - \gamma)(m_t - v_t + S/D)] \\ &= a + \sum_i d_i \gamma (m_t - p_{t-1} - v_t) - \sum_i d_i (1 - \gamma) S/D \end{aligned} \quad (41)$$

where  $a = \sum_i a_i$ .

Equation (41) states that unpredicted changes in  $m$  and  $v$  have real effects since  $p_{t-1}$  was decided on before such changes were known. In particular, if at time  $(t-1)$  the price was in long-run equilibrium, the random shocks to  $m$  and  $v$  will make output deviate from its long-run equilibrium value. A positive shock to  $m$  will increase output, as will a negative shock to  $v$ .

Given that large price changes are perceived by the monopolists to be proportionately more expensive than small price changes, the price level responds slowly to non-smooth shocks. This in turn means that the level of actual relative to desired real balances is affected by these shocks. Since the economy's demand function depends on the ratio of actual real balances to desired real balances, these shocks influence output. So, it is the lack of smoothness of the paths of money and  $v$ , and not their unpredictability, that makes nominal variables have an effect on real variables. This characteristic sets this model apart from the traditional equilibrium business cycle models.



$V$  is related to the household's desire to save. When  $v$  goes up households desire higher money balances and will save more (i.e. will consume a smaller fraction of their income at the original price level). This increase in the desire to save has no reason to be contractionary since a simple change in the price level can provide people with the new desired amount of wealth at full employment. However, if prices are slow in responding to the shock, this increase in the desired wealth holdings at full employment will indeed be contractionary.

Not only will shocks to  $m$  and  $v$  (two "nominal" variables) have real effects but these effects will in general persist. That is, a positive deviation of output from its long-run equilibrium value will in general be followed by a smaller positive deviation of output from its long-run equilibrium value. Let the long-run equilibrium value of the output index be  $q^*$ . This is the value of output that is consistent with a price level  $p_t^*$  and is given by (29) together with (27):

$$q^* = a - \sum_i d_i S/D \quad (42)$$

To check that positive deviations of  $q$  from  $q^*$  are expected to be followed by further positive deviations I will compute the expectation of the deviation of  $q$  from  $q^*$  conditional on the realizations of the previous period. It will turn out that this conditional expectation will simply be equal to the previous deviation multiplied by  $\gamma$ .

$$q_t - q^* = \sum_i d_i (m_t - v_t - p_t + S/D) \quad (43)$$

$$E_{t-1}(q_t - q^*) = \sum_i d_i (m_{t-1/t} - E v_{t-1/t} - E p_{t-1/t} + S/D)$$

$$\begin{aligned}
E_{t-1}(q_t - q^*) &= \sum_i d_i [m_{t-1} - v_{t-1} - \gamma p_{t-1} - (1-\gamma)(m_{t-1} - v_{t-1} + S/D)] \\
&\quad \sum_i d_i \gamma (m_{t-1} - v_{t-1} - p_{t-1} + S/D) \\
&= \gamma (q_{t-1} - q^*) \quad . \quad (44)
\end{aligned}$$

To show this I have used the fact that  $m$  and  $v$  follow random walks as well as the fact that the expectation of  $p$  is linear in money,  $v$  and the random components. The persistence of the effect of nominal shocks on output is due to the absence of full adjustment of the price level to its long-run value within one period. Instead the price level adjusts geometrically to its long-run level and therefore the adjustment of output towards  $q^*$  is also geometric.

The fact that independent identically distributed disturbances affecting the money stock and the taste for real money balances lead to autocorrelated responses in output is also a characteristic of the Lucas (1975) model. This characteristic is a requirement for the existence of an "endogenous" business cycle. That is, a model with this feature generates cyclic behavior in output as a response to noncyclical shocks.

When  $(m-v)$  follows a random walk only its past and present innovations affect output. More generally, as long as both  $(m-v)$  follows an ARMA process whose autoregressive representation has a unit root and the first difference of  $(m-v)$  follows a stationary process; only the innovations of the process will influence the deviation of  $p_t$  from  $p_t^*$ .

Case 2     m and v follow processes whose first difference is stationary

Let  $(m-v)$  follow a stochastic process that can be written as:

$$(m_t - v_t) = (m_{t-1} - v_{t-1}) + \sum_{i=0}^{\infty} x_i \varepsilon_{t-i} \quad (45)$$

where  $\{\varepsilon_t\}$  is a sequence of independently identically distributed normal variates with mean zero, the  $x_i$  coefficients whose sum is bounded and  $x_0$  is normalized to one.

These assumptions ensure that the innovation at  $t$ ,  $\varepsilon_t$  will have a bounded effect on all future values of  $(m - v)$ .<sup>10/</sup>

In this case the price level can be written as:

$$p_t = m_t - v_t + S/D - \sum_{i=0}^{\infty} w_i \varepsilon_{t-i} \quad (46)$$

where the coefficients  $w_i$  are computed by substituting (45) into (38), both for  $(m_{t-1} - v_{t-1})$  and for the expected value at  $t$  of  $(m_{t+k} - v_{t+k})$ . From (29), it follows that output at  $t$  is equal to:

$$q_t = q^* + \sum_i d_i \left( \sum_{i=0}^{\infty} w_i \varepsilon_{t-i} \right) \quad (47)$$

and the deviations of output from its long-run value depend only on the history of the innovations.

The  $\varepsilon$ 's also generate autocorrelated fluctuations in relative prices. By (18) and (14), those firms whose  $d_i$  is large, adjust their prices mostly towards the present and future levels of money balances, while those whose  $d_i$  is small, use mainly the price levels as their target. In other words, the stronger is the effect of real money balances on the demand for good  $i$ , the faster is firm  $i$ 's response to monetary shocks. Therefore, at  $t$ , those firms with a large  $d_i$  respond more quickly to any shock  $\varepsilon_t$  than do those firms whose  $d_i$  is small.

When  $(m-v)$  follows a process like (45), the long-run equilibrium price level changes, from one period to the next, by a bounded sum of innovations. Similarly, the expectation of future long-run equilibrium price levels changes by a different bounded sum of innovations. Hence, the price level only has to adjust to the history of the innovations. The absence of full adjustment of the price level to these innovations is what has an effect on aggregate output. The  $\varepsilon$ 's make the path of  $m$  jagged; but, the price level follows a smooth path due to the costs associated with price changes. Therefore, the  $\varepsilon$ 's affect real money balances and output.

Models of the type of Lucas (1975) obtain an equation similar to (47) for the stochastic process governing output. There, the  $\varepsilon$ 's are unperceived and become confused with profitable opportunities. Instead the deterministic components of the money supply process (like those that result from an active monetary policy) have no effect on output. The arguments that lead to an equation like (47) are therefore quite different. In fact, monetary policy has an effect on output in the model of this paper as will be shown in chapter III.

I now proceed to study another jagged path for  $(m-v)$  which, by virtue of being jagged, makes output deviate from  $q^*$ .

### Case 3 The effect of anticipated change in the future of $m$

Suppose that the money stock and  $v$  are known with certainty to remain constant until time  $N$ . From  $N$  on the money stock will be equal to the money stock at time zero multiplied by  $e^{\mu}$  where  $\mu$  is taken to be positive for expositional purposes. This information is revealed at time zero. Suppose, for simplicity, that at time minus one the price level

was such that output was at  $q^*$ . Then, using equation (38) the aggregate price levels will be:

$$p_o = m_o - v_o + S/D + \frac{D}{\delta \rho c} \sum_{j>N} \left(\frac{1}{\delta}\right)^j \mu = m_o - v_o + S/D + \frac{D}{\delta \rho c(1 - (1/\delta))} (1/\delta)^N \mu$$

$$= m_o - v_o + S/D + (1 - \gamma)(1/\delta)^N \mu \quad (48)$$

and

$$p_t = m_o - v_o + S/D + (1 - \gamma)(1/\delta)^{N-t} \frac{1 - (\gamma/\delta)^{t+1}}{1 - (\gamma/\delta)} \mu \quad t = 0, 1, 2, \dots, N. \quad (49)$$

Equation (48) makes it clear that the price level starts rising at time 0 even though the expansion in the money supply is due at time N. Therefore the announcement of the expansionary policy is contractionary, if believed, since it induces a decline in real money balances. Until the time the expansion in money actually occurs output will therefore be below  $q^*$ . Whether output will be above  $q^*$  on the day of the monetary expansion is still a question. To see that it can be answered in the affirmative it suffices to look at equation (49) with  $t$  replaced by  $N$ .

$$p_N = m_o - v_o + S/D + \mu(1 - \gamma) \frac{1 - (\gamma/\delta)^{N+1}}{1 - (\gamma/\delta)} \quad (50)$$

The larger  $N$  is, the larger the price level will be at the time the expansion actually takes place. Also, output will be at  $q^*$  at time  $N$  if and only if the coefficient of  $\mu$  in equation (50) is unity. I will show that the coefficient of  $\mu$  in (50) is less than unity even when  $N$  is large enough to make  $(\gamma/\delta)^{N+1}$  negligible.

$$\gamma\delta > \gamma$$

Since  $\delta$  is larger than one. Therefore:

$$-\gamma\delta < -\gamma$$

Therefore:

$$\delta - \gamma\delta < \delta - \gamma$$

Therefore:

$$\frac{(1 - \gamma)\delta}{\delta - \gamma} < 1$$

which states that the coefficient of  $\mu$  in equation (50) is smaller than one.

The monopolists try to spread the losses from inevitable price changes over time. In equilibrium some of these costs are incurred after  $N$ .

#### Case Four A constant rate of growth of $m$

Next I consider situations in which the money supply grows at a constant rate. The main results of this section are that in the steady state prices will grow as fast as the money stock and that the existence of a positive rate of growth of the money stock raises output above  $q^*$  if there is a positive rate of discount.

The dynamics of the money stock will be described by:

$$m_t = m_0 + \lambda t \quad (51)$$

The base level  $m_0$  could be following a random walk as before. However, I will concentrate on the certainty case to sharpen the results. Therefore I will assume that  $v$  is also fixed. In this case equation (38) which describes the dynamics of the price level reduces to:

$$p_t = p_{t-1}\gamma + (1 - \gamma)(m_0 + \lambda t - v_0 + S/D) + \frac{D\lambda}{\delta\rho} \sum_j \left(\frac{1}{\delta}\right)^j \cdot j \quad (52)$$

$$p_t = \gamma p_{t-1} + (1 - \gamma)(m_0 + \lambda t - v_0 + S/D) + \frac{D\lambda}{\delta\rho c} \frac{1/\delta}{(1 - 1/\delta)^2}$$



Multiplying the numerator and denominator of the last term by  $\delta^2$  and making use of (37) one obtains:

$$p_t = \gamma p_{t-1} + (1 - \gamma) (m_o - v_o + \lambda t + S/D + \frac{\lambda}{\delta - 1}) \quad (53)$$

This is simply a nonhomogeneous difference equation whose solution is:

$$p_t = \gamma^t p_o + (1 - \gamma^t) (m_o - v_o + S/D + \frac{\lambda}{\delta - 1} - \frac{\gamma \lambda}{1 - \gamma}) + \lambda t \quad (54)$$

where  $p_o$  is the price level at the time the money supply starts to grow. Note that in the steady state in which  $\gamma^t$  is essentially zero, the price level grows at the rate  $\lambda$ , the rate at which the money supply grows. Therefore, in the steady state real money balances and output will be constant.

In the steady state real profits must be constant. This requires that both  $(p_t - p_t^*)$  and  $(p_t - p_{t-1})$  be constant. Since  $p_t^*$  grows at the same rate as the money supply, this condition requires that prices grow at  $\lambda$ , the rate of monetary expansion.

In the steady state, the price charged by firm  $i$  will be given by:

$$p_{it} = \lambda t + (1 - \alpha) [s_i + (1 - d_i) (S/D - v_o + \frac{\lambda}{\delta - 1} - \frac{\gamma \lambda}{1 - \gamma} + \frac{\lambda}{\beta - 1} - \frac{\alpha \lambda}{1 - \alpha}) + m_o]$$

Therefore, relative prices will be constant in the steady state.

For the level of output to be  $q^*$  in the steady state, the price level would have to be:

$$p_t^* = m_o - v_o + S/D + \lambda t. \quad (55)$$

In the steady state the comparison between (54) and (55) hinges on the comparison between:

$$\frac{1}{\delta - 1} \quad \text{and} \quad \frac{\gamma}{1 - \gamma}$$

Multiplying both these terms by  $(1 - \gamma)(\delta - 1)$  which is a positive number does not change the relationship between them. This leads to the comparison of 1 with  $\delta\gamma$ . From the argument that led to (37),  $\delta\gamma$  is simply equal to  $1/\rho$  which is usually greater than one. Therefore, as long as there is a positive discount rate, the first term of the two I have been comparing will be smaller. The price level under a constant rate of monetary injection will be lower than the price level which would prevail in the absence of any costs to changing prices. A lower price level is tantamount to a larger value for the index of output given the demand equation (29). Therefore, a larger rate of inflation will lead to a larger level of output in the steady state if there is a positive discount rate.

The intuition behind this mechanical argument is simple. Suppose that at time minus one the prices were at  $p^*$  and output at  $q^*$ . Then at time zero the monopolists know that the money stock has increased by  $\lambda$  percent and have to decide on a price. They could increase their prices by  $\lambda$  percent; then their profits from operations (revenues from sales minus production costs) would remain maximal and constant in real terms. As long as they kept increasing their prices by the same percentage as the rate of expansion of the money supply their profits from operations would remain constant.

Instead the monopolists could increase their prices by slightly less in the early stages. This would decrease their costs due to price changes which are increasing in the magnitude of the price change. Instead the monopolists would receive lower real profits from operations.

This loss in profits from sales would continue until infinity given that it would never be optimal to increase prices by more than  $\lambda$  in any given period. If the discount rate is positive, the infinite stream of losses from sales that results from increasing prices by less than  $\lambda$  in the early stages is smaller than the stream of benefits that accrue from smaller changes in prices during the period that gets counted most heavily in the present value calculation. If the discount rate is equal to zero,<sup>11/</sup> any change in the level of real profits from operations which will be incurred forever will have an infinite present value and it will never be advantageous for the monopolist to increase his prices by less than the rate of monetary expansion. Therefore, prices go up by  $\lambda$  even the first period.

One might argue that, if people were certain that prices were rising by  $x$  percent a day, there could not be any costs associated with price changes due to their unfavorable effect on demand. However, the fact that people know that on average prices are rising by a certain amount a day is not the same as people knowing the rate of growth of any particular price over a particular interval of time. It is this sort of situation that I try to capture by computing equilibria with costly price adjustment. Admittedly the argument is more persuasive when there is uncertainty about the rate of inflation.

I now introduce uncertainty about the future rate of inflation while preserving the smoothness of the money supply process. Let the rate of growth of the money supply follow a random walk. The monopolists observe the current rate of monetary expansion and expect it to remain constant. If the discount factor  $\rho$  is equal to one and the initial situation is one of long-run equilibrium, they raise their prices at the current rate of monetary ex-

pansion. Therefore, the random changes in the rate of growth of the money stock have no effect on output. Output remains at  $q^*$  once the effect of the initial conditions has worn off.

This result differentiates this model from those, like Lucas', in which the business cycle is due to the unobservability of aggregate variables. In those models random changes in the growth rate of money affect output.

This section has presented two mechanisms through which the stochastic process of the money stock affects output. The first and most important one is that the price level follows a smooth path even when the money stock doesn't. The path of output doesn't just depend on the jumps in  $m$  and  $v$ ; it also depends on when these jumps are perceived relative to when they are realized. The second mechanism involves the discount rate. A positive discount rate leads to an increase in output as a response to a larger rate of monetary expansion.

#### V. The Labor Market

So far, only goods were used as inputs into the production of goods. This assumption was made mainly for simplicity since the results of this chapter can be extended to economies in which labor is a factor of production. This section proves that eq. (38) also describes the dynamics of an economy in which changing prices is costly and in which labor is a factor of production. Two types of labor markets are considered. In the first there is an economy-wide competitive labor market. In the second each firm is a monopsonistic buyer of its own type of labor.

These models also imply that the real wage (or average real wage) moves procyclically as it indeed seems to do in the U.S.

In this section the production function for good  $i$  is given by:

$$Q_{it} = H_i N_{it}^g \prod_{j=1}^n F_{ijt}^{h_j} \quad (56)$$

specific constant while  $g$  and  $h_j$  are fixed parameters. First, I determine the value of the wage rate when the labor market is competitive.

If the firms can hire any quantity of labor at the current nominal wage  $W_t$ , the minimum cost of producing  $Q_{it}$  is given by:

$$C(Q_{it}) = U_i (W_t^g \prod_{j=1}^n P_{jt}^{R_j})^{1/f} Q_{it}^{1/f} \quad (57)$$

$$\text{Where } f = g + \sum_{j=1}^n h_j \quad \text{and} \quad U_i = H_i g \prod_{j=1}^n h_j^{1/f} \quad (58)$$

Therefore, the demand for labor by firm  $i$ , which is simply the derivative of the cost function with respect to  $W_t$ , is given by:

$$N_{it} = \frac{g U_i}{f} Q_{it}^{1/f} W_t^{\frac{g-f}{f}} \prod_{j=1}^n P_{jt}^{h_j/f} \quad (59)$$

More labor is demanded as either the nominal wage falls, the price level rises or profit maximizing output rises. A fortiori the demand for labor is negatively related to the real wage  $R_t$  such that:

$$R_t = W_t/P_t = W_t / \prod_{j=1}^n P_{jt}^{(h_j/\sum h_j)} \quad (60)$$

Using the definition (60) the demand for labor by firm  $i$  becomes:

$$N_{it} = \frac{gU_i}{f} Q_{it}^{1/f} R_t^{-(\sum h_j/f)}$$

In this section I assume that the parameter  $d_i$  is constant across firms and is equal to  $d$ . Aggregating over firms, one obtains the amount of labor employed at  $t$  as:

$$N_t = \frac{g}{f} R_t^{-(\sum h_j/f)} \left( \frac{M_t}{P_t V_t} \right)^{d/f} \sum_{i=1}^n U_i A_i \left( \frac{P_{it}}{P_t} \right)^{-b_i/f} \quad (61)$$

By approximating the weighted average of relative prices by a time invariant constant, one obtains:

$$N_t = \Lambda_1 \left( \frac{M_t}{P_t V_t} \right)^{d/f} R_t^{-(\sum h_j/f)} \quad (62)$$

where  $\Lambda_1$  is a constant.

The labor supply function is assumed to be:

$$N_t = \Lambda_2 R_t^w \quad (63)$$



where  $\Lambda_2$  and  $w$  are constants.

Then, equilibrium in the labor market requires that:

$$R_t^{w + \frac{\sum h_j}{f}} = \frac{\Lambda_1}{\Lambda_2} \left( \frac{M_t}{P_t V_t} \right)^{d/f} \quad (64)$$

As real money balances grow, the level of output grows. This induces a larger amount of labor to be demanded at each real wage. Therefore the equilibrium real wage grows along the supply of labor function (63).

I now show that the equilibrium with costly price adjustment is described by (38) when the real wage is determined by (64).

In the absence of costs to changing prices firms would charge:

$$P_{it}^* = \Theta_i U_i P_t R_t^{g/f} Q_{it}^{1/f-1} \quad (65)$$

where:

$$\Theta_i = \frac{b_i - (b_i - d) h_i / \sum h_i}{f(b_i - 1 - [(b_i - d) h_i / \sum h_i] + h_i / f)}$$

Therefore, using (64)

$$P_{it}^* = p_t + s_i + \frac{d}{1+b_i} \left( \frac{1-f}{f} + \frac{1}{fw + \sum h_j} \right) (m_t - v_t - p_t) \quad (66)$$

where

$$s_i = \left[ \frac{(1-f)a_i}{f} + \theta_i + u_i + \frac{f}{fw + \sum h_j} (\lambda_1 - \lambda_2) \right] / 1 + b_i$$

Real profits at  $t$  can be approximated by:

$$\Pi(p_{it}) = \Pi(p_{it}^*) - k_i(p_{it}^* - p_{it})^2$$

$$k_{it} = U_i R_t^{g/f} Q_{it}^* \frac{1-b}{f} [(b_i - (b_i - d)h_i / \Sigma h_i)(\frac{1}{f} - b_i(b_i - d)(h_i / \Sigma h_i)(\frac{1}{f} - 1))]$$

where the variation of  $k_i$  over time will once again be neglected.

Again, assuming that  $c = c_i/k_i$  the price level at  $t$  is described by (38) as claimed with:

$$D = d \left[ \sum_{i=1}^n \left( \frac{1-f}{f} + \frac{1}{fw + \Sigma h_j} \right) h_i / (1 + b_i) \right] / \Sigma h_i$$

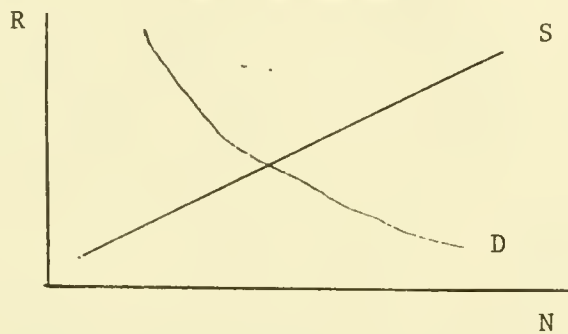
$$S = \frac{\Sigma h_i s_i}{\Sigma h_i}$$

As seen in Section IV, increases in  $m$  are expansionary because the price level does not immediately reach its long run value. Therefore, increases in the money stock are expansionary by increasing the level of real money balances. However, as real money balances and output rise, the real wage must also rise since more labor must be employed. Therefore, the real wage will be positively related to output.

The textbook Keynesian model as exposited, for instance, in Branson [1979], the rational expectations contracting models of Fisher [1973], Taylor [1980], and the Lucas [1975] model, when interpreted as including a labor market have a common feature. In these models the real wage must move countercyclically. These models explain the business cycle

by assuming that workers are unable to observe the current real wage or cannot supply their most desired quantity of labor at the current real wage. In either case an increase in the money stock raises the price level, reduces the real wage paid by firms and hence encourages firms to hire more labor and increase output. Output increases only because the real compensation of workers falls.

This can be seen in the standard partial equilibrium labor market diagram:



In the traditional models of the business cycle, changes in the money stock move the supply curve for labor, and output is determined along the demand curve for labor. A lower real wage induces more employment and higher output through the production function.

In the model proposed by this thesis, output changes when firms prefer output changes to price changes. That is, the demand for labor shifts in response to monetary shocks. Therefore, the real wage moves procyclically along the supply curve for labor.

The real wage does move procyclically in the U.S. Therefore, the model of this thesis seems to describe the U.S. economy better than the more traditional macroeconomic models.

I now show that these results apply also to a model in which each firm is a monopsonistic buyer of its type of labor. Let the supply of labor to firm  $i$  be given by:

$$N_{it} = \Lambda_{3i} R_{it}$$

where  $\Lambda_{3i}$  is a firm specific constant. Then the minimum cost of producing  $Q_{it}$  is given by:

$$C(Q_{it}) = U_i Q_{it}^{1/e} p_t \quad (67)$$

where

$$U_i = 2 \left( \sum_{j=1}^n h_j + \frac{g}{z\Lambda_{3i}} \right) \left[ H_i \left( \frac{g}{2\Lambda_{3i}} \right)^{g/2} \left( \prod_{j=1}^n h_j \right)^{1/e} \right]$$

$$e = \frac{g}{2} + \sum_{j=1}^n h_j$$

Therefore, as long as  $e$  is smaller than one, a requirement consistent with production functions exhibiting constant returns to scale, (38) describes the dynamics of this economy with:

$$\theta_i = \frac{b_i - (b_i - d)h_i / \sum h_i}{e [b_i - 1 - (b_i - 1 - d)h_i / \sum h_i]}$$

$$k_i = \frac{U_i}{e} Q_i^* \frac{1-e}{e} \{ (b_i - (b_i - d)h_i / \sum h_i) [1 + (e-1)(b_i - (b_i - d)h_i / \sum h_i)] \}$$

$$s_i = \left( \frac{(1-e)a_i}{e} + \theta_i + u_i \right) / (1 + b_i)$$

$$S = \left( \sum_i s_i h_i \right) / \sum_i h_i$$

$$D = d \left( \sum_i \frac{(1-e)h_i}{e(1+b_i)} \right) / \sum_i h_i$$

Next I compute the real wages paid to workers. The demand for good  $j$  by firm  $i$  is given by:

$$F_{ijt} = U_i Q_{it}^{1/e} \left( \frac{P_t}{P_{jt}} \right) \frac{h_j}{\sum h_j} \quad (68)$$

Therefore, using the production function, it is true that:

$$Q_{it} = \frac{H_i U_i \prod_{j=1}^n h_j^{\frac{\sum h_j}{\sum h_j}}}{\sum h_j} Q_{it}^{(\sum h_j/e)} N_{it}^g$$

Therefore, the quantity of labor employed by firm  $i$  at time  $t$  is:

$$N_{it} = K Q_{it}^{1/2e} \quad (69)$$

where

$$K = \left[ H_i U_i \prod_{j=1}^n h_j^{\frac{\sum h_j}{\sum h_j}} / \left( \sum h_j \right)^{\frac{(\sum h_j)}{g}} \right]^{-\frac{1}{g}}$$

Therefore, as aggregate output rises, the  $Q_{it}$  will on average rise, total employment will increase and the average real wage will rise as before.

## VI. Conclusions

This paper has presented a model of an economy that is characterized by fluctuations in aggregate output as responses to fully perceived "nominal" shocks.

Both the textbook Keynesian model and the Lucas model are based on a varying supply curve of labor over the business cycle. At certain time, those with relatively high prices, workers misperceive their current return to working to be higher than usual. In these periods they work at lower real wages, they work more, and thereby increase output. Demand shocks only affect GNP by first confusing producers about their trading opportunities. It is somewhat implausible that these misperceptions can affect output by as much and for as long as is necessary to explain actual business cycles.

Instead, models in which producers are aware of their true trading opportunities, at least insofar as these are affected by the value of aggregate statistics concerning the present, seem more desirable. Also, models that imply that the real wage moves procyclically are more appealing than those that imply the opposite.

In this paper a model is presented that has both of these desirable features. It assumes that firms are price setters who perceive price changes to be costly.

These perceptions may well be reasonable if customers react unfavorably to such price changes. The equilibrium of this model when producers have rational expectations about aggregate variables has the property that the price level follows a smooth path in response to known shocks. Jagged processes for the money stock like those that appear to be relevant in the



United States generate "cycles" in output.

This model may be extended in various directions:

The goods could be assumed durable and the monopolists, if not the households, allowed to keep inventories. They would choose their level of inventories optimally taking into account both the convexity of their cost function and the cost to changing prices. The resulting equilibrium would be a joint stochastic process for output, the price level and the aggregate level of inventories. This joint stochastic process would be driven by monetary shocks as in this paper.

The firms could be subjected to firm-specific shocks. If, in addition, they were unable to observe the aggregate statistics, their inference problem when choosing prices would become similar to the inference problem faced by a Phelpsian islander. This would probably yield an even slower response of the price level to unsystematic nominal shocks.

Producers could face lower than infinite costs to rationing consumers. Then they might occasionally choose to keep their prices near their previous prices while turning away some consumers. These rationed consumers would, in turn, increase their demands in other markets. Whether there would, in such a model, ever be any rationing in equilibrium is an open question.

## FOOTNOTES

1. This "irrationality" is a property of the usual Nash equilibrium in which the price is the strategic variable.
2. It is not clear which of (5) and (6) is larger. Both the numerator and the denominator of (6) are larger. In other words, it is not clear under which regime the price charged by any given monopolist is higher. When he is concerned with nominal profits a low price reduces his nominal costs; when he is concerned with real profits a low price increases his real revenues. They would be identical if the monopolists didn't have or didn't realize they had an impact on the price level.
3. A conceivable mechanism that translates the unfavorable effect of price changes on customers into a cost to the monopolist who changes his prices is the following: Suppose people like to take time to think between the time they see a price and the moment of actual purchase. Upwards movements in prices will then be followed by a period of low demand in which people are digesting the new information and deciding whether they wish to buy at the new prices. Obviously people will only take time to think about the desirability of purchasing a particular item if they think that the probability that its price will change is low. Consumers will therefore make up their minds faster in periods of high inflation. The cost of changing prices should therefore decrease as the rate of inflation increases.
4. The curvature of the function that gives the cost of adjustment has important implications as shown by Rothschild (1971) and Folkerts-Landau (1981). In particular convex costs of adjustment like those used in

in this paper lead to gradual price changes. Meanwhile, fixed costs per price change (which are concave in the price change)

lead to abrupt and irregular changes in individual prices.

The costs of changing prices are assumed to be symmetric around zero. It would appear to be more reasonable to assume that it is costly only to increase prices. However these two assumptions lead to similar consequences in environments like the postwar U.S. in which prices only move mainly upwards.

5. An increase of all the  $a_i$ 's by a common amount  $y$  reduces equilibrium real balances by  $y/d$ . Output is therefore unaffected by such a change. A simultaneous change of all the  $a_i$ 's is indistinguishable from a change in the desire to hold real balances.
6. A model similar to this one can be used to study the rigidity that results from the monopolists being forced to set their prices before observing their demand curve. This is the rigidity focused on by Gordon and Hynes (1970).
7. This results from the existence of monopolies in the intermediate goods sector. This result is also present in a model by Hart [1980] in which workers have market power.
8. There is an issue as to how the monopolists themselves compute the expectations of the future price levels. If they use all the other monopolists first order conditions the assumption that they take prices other than their own as given is somewhat suspect. It is therefore better to think that the monopolists act as if they had somehow been informed of the stochastic process of the actual price level.

9. This paper will consider the effect of paths of  $m$  and  $v$  which are exogenous, that is unaffected by the behavior of  $p$  and  $m$ . Rotemberg [1981] also studies the scope for systematic monetary policy.
10. These assumptions rule out, for instance, the possibility that the rate of growth of  $(m - v)$  follows a random walk.
11. If the discount rate is equal to zero only the finite horizon problem of the form of (11) can be solved. However this does not change the qualitative nature of the results.

## APPENDIX TO CHAPTER II

This appendix studies the error in the computation of profits one makes when assuming that  $k_{it}$  is constant over time.

$$\text{Let } k_{it} = S\left(\frac{M_t}{p_t v_t}\right)^{g_i} \quad \text{where } g_i = \frac{d_i}{1+k_i}$$

If there were no costs to changing prices the equilibrium would have the property that  $\left(\frac{M_t}{p_t v_t}\right)$  would be constant and equal to  $N^*$ .

Now one can approximate  $k_{it}$  by:

$$\begin{aligned} k_{it} &\sim S(N^*)^{g_i} + g_i S(N^*)^{g_i-1} \left[ \frac{M_t}{p_t v_t} - N^* \right] \\ &\sim S(N^*)^{g_i} + g_i S(N^*)^{g_i-2} [m_t - p_t - v_t - n^*] \end{aligned}$$

I am implicitly assuming that firms approximate  $k_{it}$  by  $S(N^*)^{g_i}$ . This approximation would be exact in the absence of costs to changing prices when  $p_{it} = p_{it}^*$ . In the presence of these costs, however, firms will compute their profits incorrectly if they use this approximation. On the other hand the error they will make will be proportional to:

$$(p_{it} - p_{it}^*)^2 (m_t - p_t - v_t - n^*).$$

At the equilibrium with costs of changing prices  $(p_{it} - p_{it}^*)$  is of the same order of magnitude as  $(m_t - p_t - v_t - n^*)$  and therefore the error induced by the approximation is of the third order of  $(p_{it} - p_{it}^*)$ . Since terms in the third order of  $(p_{it} - p_{it}^*)$  are assumed small enough to be negligible this approximation appears reasonable near the prices firms actually contemplate charging.

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